Predicting and Pricing the Probability of Default

Alessio A. Saretto[♣]

August 4, 2004

ABSTRACT

In this paper we study how corporate bond defaults can be predicted using financial ratios and how the forecasted probability of default relates to the cross-section of expected stock returns. Using several performance measures we find that the duration model outperforms existing models in correctly classifying both *Default* and *Non-Default* firms. Using the default probabilities predicted by our model, we analyze the relation between default risk and the Fama-French distress factors, HML and SMB. We find evidence that supports the interpretation on HML as a distress related factor. Both portfolio and individual stock factor loadings are related to the estimated default probabilities. We find a negative and significant contemporaneous correlation between HML and shocks to the level of aggregate financial distress.

JEL classification: C41, G10, G33

Keywords: Default, Hazard Rate, Distress Factors

[♣]Correspondence to: asaretto@anderson.ucla.edu, The Anderson School at UCLA, 110 Westwood Plaza, Los Angeles CA, 90095. The author would like to thank Laura Frieder, Amit Goyal, Matthias Kahl, Richard Roll, Pedro Santa-Clara, Avanidhar Subrahmanyam, and Walter Torous for valuable suggestions and helpful comments. All remaing errors are the author's responsability.

It has long been argued in the empirical asset pricing literature that the cross-section of stock returns is related to risk factors associated with systematic financial distress. For instance, Chan, Chen, and Hsieh (1985) and Chen, Roll, and Ross (1986) find that a default factor, the return difference between low-grade bonds and long-term government bonds, has a significant contribution in explaining the cross-section of stock returns. According to asset pricing theory, stocks that underperform when the economy is in a high-distress state should reward the investors who hold them with higher expected returns as a compensation for bearing this non-diversifiable risk. Under these circumstances, these high returns would be partially unexplainable by a model that does not account for a distress factor, and therefore would be considered apparent mispricings.

The existence of "pricing anomalies" such as the size and book-to-market effects has in fact been widely documented. The essence of the anomalies lies in the fact that they cannot be justified by the return's covariance with the market factor. The difference in market betas cannot explain the return differential between small and large firms, and between stocks with high and low book-to-market values. In equilibrium, unless mispricings can subsist, since market risk alone does not price these stocks some other systematic risk factor must. In their seminal paper Fama and French (1993) identify two stock market factors related to size and book-to-market, SMB and HML, that in conjunction with the market factor form an impressive pricing model. In a later paper, Fama and French (1995) observe that high book-to-market firms tend to be relatively distressed, coming from a persistent period of negative earnings (this result is in part true for small firms as well), suggesting that size and in particular book-to-market capture a firm's level of financial distress.

Though the success of the three-factor model in pricing the cross-section of stock returns and portfolios formed in many ways has been reported for different time periods and countries (Fama and French (1998) and Davis, Fama, and French (2000)), we lack a complete understanding of what kind of risk (if any) the Fama and French (FF) factors price. In particular, there has been little success in showing that the factors proxy for any systematic source of risk related to financial distress. For example, Vassalou and Xing (2004) provide evidence that distress risk is priced in the cross-section and that the FF factors capture some of the default-related information.[1] On the contrary a myriad of alternative explanations has been offered: Daniel and Titman (1997) and Brennan, Chordia, and Subrahmanyam (1998) find evidence that the characteristics and not the factors loadings explain the cross-section of expected returns. According to these studies, the FF factors would price the cross-section of stock returns because they are endogenously related to the firm's characteristics and hence do not represent any sort

[1]In related papers, Liew and Vassalou (2000) and Vassalou (2003) show that the FF factors contain information related to future GDP growth.

of systematic risk, rather an idiosyncratic risk that has not been fully diversified. These paper have lead the way to the rise of alternative behavioral explanations of this rational incongruity. For example, Daniel, Hirshleifer, and Subrahmanyam (2001) present a static equilibrium asset pricing model in which "irrational mispricing" affects the cross-section of returns. They propose the size and book-to-market effects as manifestations of the persistence of this mispricing.

This study investigates the relationship between SMB, HML, and financial distress risk. It is important to notice that the existence of this relation relies on the verification of two conditions: stocks that are more likely to default should positively covary with the FF factors, and high realizations of HML and SMB returns should correspond to news about aggregate default risk. In particular, if distress risk is a systematic risk and is related to book-to-market and size, it should be the case that high book-to-market (small size) stocks underperforme low book-to-market (large size) stocks in periods when the aggregate default risk is very high.

It is therefore necessary to define measures of financial distress both at the individual (firm) and aggregate level. We characterize individual distress as the instance wherein a firm is unable to fulfill its contractual obligations. In particular, we focus on debt default, instead of firm bankruptcy, since the former in general represents an earlier instance of financial distress, while the latter constitutes only one of many possible outcomes. Additionally, debt default is better defined and more easily identifiable.[2] Since firms may have high likelihoods of default and therefore require high risk premia and yet not default in sample we cannot rely our inference on the mere occurence or non-occurence of default events. Rather we need to construct a continuous measure of expected individual distress, i.e. a conditional probability of default. The aggregate distress measure is then simply obtained by computing a value-weighted average of the individual probabilities.

The measurement of the expected default probability is critical in understanding how default risk relates to stock returns. The probability of default can be measured with a model that uses firm's financial information to obtain an indication of how likely the firm is to enter distress in the near future. Since we focus on bond default, we need a model that predicts defaults

[2] It is possible that a firm files for Chapter 11 without being in financial distress. A famous case is that of Continental Airlines, wherein Frank Lorenzo, after acquiring the company in 1982 made a "strategic" bankruptcy filing in 1983 essentially to avoid further negotiations with the airline's labor union. Weiss and Wruck (1998) describe Continental and related Eastern Airlines bankruptcy filings. In more general terms, debt default is also more relevant for public and private investors, who are interested in timing their decisions whenever possible. Managers would be interested in the correct assessment of the probability of default of the firm's obligations because, unlike the case of bankruptcy, they would still have discretion on the decision process before entering Chapter 11. Nonetheless, the creditors are the most interested in quickly resolving uncertainty about the future of the firm. For example, Kahl (2002a) argues that the long-term nature of financial distress can be interpreted as the result of the inability, on the creditors side, to identify economically viable firms from firms that should be liquidated.

as accurately as possible.[3] Forecasting precision is imperative because we use the predicted probability of default to examine the relation between distress risk and stock returns. A more informative measure will give us a more complete understanding of what, if anything, the FF factors represent.

Consequently, this paper consists of two major parts. First, we examine how corporate debt defaults can be predicted using financial information and propose a model for forecasting default. Second, we analyze whether and how this estimated measure of credit risk (the probability of default) is related to the FF factors at the firm and aggregate level.

Using data from Moody's Default Research Database as well as other hand-collected data from Dow Jones Interactive,we address the first question. In particular, we evaluate the performance of our own duration model (introduced later in the paper) compared to other models found in previous literature that attempt to predict corporate bankruptcy. We test the models on a sample of non-financial firms, which have outstanding debt, and are listed for at least one year between 1979 and 2000 on any of the three major US stock exchanges. Using several measures to gauge the accuracy of each model, we find that the duration model we propose outperforms the other models in correctly classifying both *Default* and *Non-Default* firms.

We approach the second issue using the monthly stock returns from the CRSP database and the FF factors obtained from Kenneth French's web site. We find evidence that both portfolios and individual stocks returns are related through the distress factors to the default probability measured by our statistical model. In particular, the FF factors loadings of stocks and decile portfolios sorted on the default probability are monotonically increasing. This fact provides evidence that securities that are more likely to default positively correlate with the FF factors. Further, a Fama and MacBeth (1973) two-step regression shows that the factor loadings on HML are cross-sectionally related to the default probability even after controlling for firm characteristics, such as size and book-to-market. The evidence about SMB is however not conclusive. Perhaps the most important result of this paper is the evidence of a statistically significant and negative relation between HML realizations and news about aggregate distress

[3] The prediction of the probability of default has also a considerable importance in risk management applications wherein default risk usually is referred at as credit risk. Credit risk is present in any financial contract that involves one or more payments from one contractor to another. Its importance is particularly evident in both fixed income securities, where the time and amount of payments is usually predetermined, and in all non-standardized contracts wherein transactions are not cleared in a regulated market, but are privately arranged between the counterparties. Recently, financial markets have witnessed several episodes of declaration of insolvency. Large and small countries, as well as corporations have experienced difficulties in meeting their financial obligations, sometimes triggering worldwide financial crises. These events have considerably attracted the attention of both academics and regulators. In June 1999, for example, the Basel Committee on Banking Supervision proposed a revision to the international capital accord that suggested a more prominent role for internal credit risk assessments based on the determination of he probability of default of a borrower or group of borrowers.

risk given by the shocks to the aggregate default probability. The statistical correlation is economically supported by the finding that the HML average return in years characterized by bad news about the level of aggregate default risk is about -4.5% per year.

This paper is related to a number of other studies. In the finance and accounting literature, the estimation of the probability of bankruptcy has been an important topic since the seminal works of Beaver (1966) and Altman (1968) who, by comparing matched samples of failed versus non-failed firms, show that business failures can be predicted by information contained in financial ratios. Other important contributions in that field are: McFadden (1976), Altman, Haldeman, and Narayanan (1977), Santomero and Visno (1977), Ohlson (1980), Zmijeski (1984), Lo (1986), Queen and Roll (1987), and Shumway (2001). Our model differs from those cited above in two essential dimensions: it allows time-varying regressors and flexibility in the specification of the hazard function. Following Sueyoshi (1995),[4] we derive and estimate a simple piecewise constant hazard specification. Contrary to the existing literature, we find strong evidence of a duration effect on the probability that a firm incurs a corporate default event.

Other studies have tried to link measures of bankruptcy risk to stock returns. Dichev (1998) finds that bankruptcy risk is not rewarded by higher returns and concludes that a distress factor cannot be at the origin of the size and book-to-market effects. In particular, he finds that portfolios formed on the basis of a distress measure, whether Altman's Z-score or Ohlson's O-score, have returns inversely related to bankruptcy risk: a high probability of default is associated with low average returns. After examining the cross sectional relation between stock returns and bankruptcy measures, as well as size and book-to-market, he concludes that the fact that low-bankruptcy-risk firms outperform high-bankruptcy-risk firms can only be explained by a mispricing argument. Griffin and Lemmon (2002), measuring bankruptcy risk by the Ohlson score, find that the low return of high-bankruptcy-risk firms is driven by low book-to-market stocks with extremely low returns. They attribute these very low returns to mispricing due to a high degree of information asymmetry proxied by low analyst coverage. Vassalou and Xing (2004) use the distance to default implied by the Merton (1974) model to conclude that the the size and book-to-marekt effects exist only in the quintiles defined by high default risk stocks. They also provide evidence that distress risk is priced in the cross-section and that the the FF factors capture some of the default-related information. Contrary to these papers, we use a different default probability model and we focus on testing the hypothesis that the Fama and

[4]Sueyoshi (1995) examines the relationship between grouped hazard specifications and binary response models, finding that the likelihood of a particular observation on a group proportional hazard model corresponds to the probability of observing a series of binary outcomes. However, a proportional hazard model does not exactly correspond to a simple pooled probit or logit specification. Indeed, it allows an easily estimable form of time-varying and covariate-dependence in the conditional survival probability.

French (1993) factors are related to a (in particular, *our*) default risk measure, rather than trying to explain directly the size and book-to-market effects. First we try to establish a connection between returns and the probability of default, and between the probability of default and factor loadings. We anticipate that expected returns and default risk are related. A high probability of default should cause a stock to have high loadings on the HML and SMB factors, thus delivering high expected returns which compensate the investor for holding default risk. Second we try to verify if the FF factors are related to aggregate default risk. We find evidence in support of the interpretation of HML as a distress related factor.

The rest of this paper is organized as follows: in Section I, we describe the sample and the variables that enter each forecasting model. We review the methods used in the literature and derive the duration model in Section II. We discuss estimation and prediction results in Section III. Our asset pricing tests are explained in Section IV. Section V concludes.

I. Data

A. Sample

Our sample contains all firms present in both the CRSP and COMPUSTAT databases between 1979 and 2000.[5] We choose 1979 as the starting point because of the 1978 change in the bankruptcy law. We employ only non-financial firms that are traded in one of the three major markets with some issued debt and at least one million dollars in total assets.The sample we obtain consists of 7,282 firms, yielding a total of 48,967 firm-year observations. If a firm is listed on CRSP between 1975-92 and in COMPUSTAT between 1987-95, it enters our sample only between 1987 and 1992. Its duration, defined as the number of years since it became a public firm (present in CRSP), however, is 18 years.

For the purpose of this study, we use bond-related defaults as our "default event". We consider any missed payment of interest or principal, any violation of a covenant, any attempt to restructure, or any declaration of insolvency as a default event. We hand-collect 1,545 events over the sample period from *Moody's database*, *COMPUSTAT's research file and footnotes*, *CRSP's delisting codes*, and *Dow Jones Interactive*.[6] Not all events can be used in our analysis, as many

[5] There are two basic sampling schemes we can adopt: stock sampling and flow sampling. Flow sampling suggests that we choose a date and include in our sample only those firms which, according to our general criterion, become observable, i.e. flow into the sample. Stock sampling, on the other hand, allows us to consider all firms which meet the observability criterion at any particular date. We merge these methods by including all firms listed in 1979 and the following years.

[6] Most of the defaults were found in Moody's. Since this dataset tends to leave unmentioned issues which are not rated by Moody's, we complete our collection by adding firms with a CRSP bankruptcy delisting code (572 and 574) and firms that in COMPUSTAT footnote 35 have code 02 (filing for Chapter 11) or 03 (Chapter 7).

of them are related to either financial or foreign firms. We exclude these observations because they are subject to different bankruptcy laws. The unavailability of accounting information for a firm in a specific year further and dramatically reduces the sample size. To mitigate the effects of a small sample, we include firms that default up to two years after the stock is delisted. We assign the delisting year to the year of default. After these adjustments, the default sample contains 319 firms.

B. Variables

The most well-cited bankruptcy models in the literature are Altman (1968), Ohlson (1980), Zmijeski (1984), and Shumway (2001), all of which attempt to account for the aspects of the firm that may play a role in leading to a corporate default. Economic viability, growth possibility, liquidity, and financial structure are all important determinants of financial deterioration and, therefore, can be used as conditioning variables.

The Altman (1968) Z-score model is based on five ratios: working capital to total assets [WC/TA], retained earnings to total assets [RE/TA], operating income after depreciation to total assets [EBIT/TA], market value of equity to total liabilities [MV/TL], and sales to total assets [S/TA]. Ohlson (1980) estimates his O-score model using the log of total assets [log(TA)], a measure of leverage [TL/TA], the ratio of current assets to current liabilities [CA/CL], the ratio of net income to total assets [NI/TA], funds provided by operations divided by total liabilities, and two measures of past income performance: [INTWO] and [CHIN]. Zmijeski (1984) parsimoniously selects only three of the above variables: TL/TA, CA/CL and NI/TA. Shumway (2001) estimates his bankruptcy model using accounting variables from Altman (1968) and Zmijeski (1984), as well as other market variables, such as the past-year stock performance [Diff-Ret], the idiosyncratic standard deviation of returns [Sigma], a measure of market capitalization [Rel-Size], and the firm's age [Age]. For our models, we introduce new variables that have a very intuitive relation with corporate default: the coverage ratio [COV] and Tobin's [Q].

A detailed description of how the variables are constructed and of the rationale for their use is reported in Appendix A. Summary statistics are reported in Table I. Results for the *Non-Default* firms are presented in the Panel A of the table. While in most cases the mean and median are reasonably close, the minimum and maximum are highly variable, suggesting the presence of significant outliers. Panel B reports statistics for our *Default* sample. For fifteen of the eighteen variables, the difference between the means of the two groups is statistically different from zero at the 1% significance level (WC/TA , S/TS, and CA/CL are the exceptions). Compared to the

Moreover, we try to expand our collection of default events by searching Dow Jones Interactive with the following keywords "bond and default", "bond and interest payment" and "bond and principal payment".

average *Non-Default* firm, the average *Default* firm is worth approximately 100 million dollars less in TA, is 1 year younger, and is less profitable (-3% in operating performance). Additionally, it has accumulated several years of negative performance, as indicated by a negative RE/TA and a higher mean value of INTWO. It is more leveraged (66% vs. 57%) and has a difficult time covering interest expenses, which results in a lower market valuation measured by Q (1.27 vs. 1.57), a higher idiosyncratic standard deviation (13% vs. 10%), and a negative past market performance (-9.7% vs. 1.5%).

II. Statistical Models of Default

A. *Existing Models*

In the existing literature, researchers have forecasted bankruptcy using different methods such as multivariate discriminant analysis and single and multi-period logit models. MDA is first applied by Altman (1968) and is widely used in several later studies and for different applications. This method attempts to construct a function that linearly combines financial ratios into a single indicator variable, which is monotonically decreasing in the probability of failure. The researcher then assigns any firm to one of the two groups, *Default* and *Non-Default*, discriminating on the basis of the indicator, often called the Z-score.

A logit model is a latent variable model, wherein the riskiness of a firm (y^*) is unobservable; we observe only whether a firm defaults.[7] First applications of this technique to the study of corporate bankruptcy are, for example, Santomero and Visno (1977) and Ohlson (1980). These are considered *static* single-period models because every firm enters the sample with only one observation. More recently, Shumway (2001) shows how a multi-period logit model, wherein each firm enters with multiple observations, can be used to predict bankruptcy.

B. *Duration Model*

Like the above models, duration models can be utilized to analyze how a firm's characteristics can explain the amount of time a firm spends in a particular state of nature, characterized by the firm's ability to fulfill its debt obligations. We call this state *financial health*. When the firm is not able to repay its creditors, it defaults, and enters another state that we denote *financial distress*. Firms enter and emerge from financial distress in different ways and with different

[7] A latent variable is linear in some firm characteristics and in a logistic-distributed random shock, originating a probability of default with a non-linear form. It can be defined by two equations $y^* = \beta x + \epsilon$ and $y = 1_{(y^*>0)}$, from which it is immediate to derive the probability of default as $P(y=1|x) = P(y^*>0|x) = P(\epsilon > \beta x) = \frac{e^{xb}}{1+e^{xb}}$. If β is positive, the default probability is monotonically increasing in the value of the regressor.

outcomes. Kahl (2002b), for example, follows a sample of distressed firms through their history, and finds that only about one-third of them survive as independent firms, while the others are either acquired or disappear. As a statistical model that takes into account all possible outcomes for a firm is beyond the scope of this study, for a given firm, we focus on predicting only the first case of financial distress, namely corporate default. We disregard what happens afterwards.

In a duration model, time T is a continuous random variable that begins when the firm's stocks are first traded and ends when a default event occurs. T has respective density and distribution functions $f(t)$ and $P(T < t) = F(t)$.

Using simple manipulations it can be shown that the survival function $S(t) = 1 - F(t)$ can be expressed in terms of only the hazard function $\theta(t)$ (the instantaneous rate of default per unit of time) obtaining

$$S(t) = e^{-\int_0^t \theta(s)ds}. \tag{1}$$

The hazard function $\theta(t)$ can assume any functional form with respect to time and individual characteristics, but a common choice is the exponential form:

$$\theta(t) = e^{\beta x(t)}, \tag{2}$$

where $x(t)$ is a vector that may or may not include time/age in the set of regressors, and β is a vector of paramteres to be estimated.

As we can see from Equation (1) computing the survival rate involves integrating the hazard rate. This potentially could be a problem if the regressors vary continuously with time. The vector $x(t)$ is composed of balance sheet ratios and market variables. Since balance sheets become public at fixed intervals of time, $x(t)$ is constant over the interval $[t-1, t)$, between the realization of one balance sheet and the next. Given this, we assume that information is realized at the beginning of each period and no more information is available until the next period.[8] Finally, for tractability, we impose $x(0) = 0$. We also assume that defaults can happen only at the moment before the release of new information. We find that the hazard rate is constant throughout the interval $[t-1, t)$, and is therefore a step function over the domain of T. Equation (1) becomes

$$S(t \mid \beta, x(1), \dots, x(t)) = e^{-\sum_{k=1}^{t} \theta(k)} \tag{3}$$

[8]While it is reasonable to think that it could be the case for accounting variables, this assumption does not strictly hold for market variables. Even with not-perfectly efficient markets new information is incorporated into prices every day.

and the likelihood function of the hazard rate model can be written as

$$\ell = \prod_{i=1}^{N} [S_i(y_i - 1 \mid \beta, x(1), \ldots, x(t_i - 1)) - S_i(y_i \mid \beta, x(1), \ldots, x(t_i))]^{d_i} \left[\frac{S_i(y_i \mid \beta, x(1), \ldots, x(t_i))}{S_i(y_i \mid \beta, x(1), \ldots, x(e_i))} \right]^{1-d_i},$$

where $d_i = 1$ if the firm defaults by year t_i, $y_i = \min(t_i, c_i)$ is the duration, t_i is the uncensored duration, c_i is the number of years we observe the firm before it is right censored (see Appendix B), and e_i is the number of years firm i exists before we observe it. The interested reader can find a description of how to derive the likelihood function in the Appendix.

III. Predicting Default

A. The Estimates

We use the observations between 1979 and 1996 as the estimation sample and those remaining as the out-of-sample validation group. We estimate the Altman (1968) model by discriminat analysis, the Ohlson (1980), Zmijeski (1984), and Shumway (2001) models using a logit model, as well as two specifications of the new duration model introduced in Section II.B. The first specification considers only balance sheet ratios. We estimate this model not only because it is important to understand the value of accounting information, but more importantly because it gives us a tool that could be used in credit-risk evaluation of companies that are not publicly traded, and for which we lack any kind of market information. Consistent with practitioners, we call this model the *Private* firm model. The second, henceforth *Duration*, uses the significant conditioning variables from the other models.

Parameter estimates and t-statistics are presented in Table II. The parameters of Altman's model [Alt] are substantially different in our time period as compared to the original 1968 study (which is still used by practitioners and academics). WC/TA and EBIT/TA now have greater weight in the score function (1.2 and 3.3 vs. 1.8 and 6.1 respectively). Surprisingly, RE/TA and S/TA enter with a negative sign.[9] The Ohlson model [Ohl] relies on a substantially different set of variables. CA/CL and log(TA) are only marginally statistically significant. CA/CL should have a negative effect, but it has a positive coefficient. On the other hand, the univariate analysis suggests that the difference in means between CA/CL for the two groups is statistically

[9] A high z-score is associated with a good standing financial position. Thus, while a low or negative score indicates a critical position, we should expect all the five variables to have positive coefficients. Other researchers have found a negative coefficient for S/TA, for example Begley, Ming, and Watts (1996), but it seems implausible that default risk decreases with past earnings, unless of course RE has a negative value. The average RE/TA value is indeed negative for the *Default* group which, combined with large negative outliers in the data, could be driving the result.

insignificant. The Ohlson model indicates that TL/TA, INTWO, and CHIN are also related to corporate defaults. As mentioned above, the Zmijewski model [Zmi] includes variables also present in the Ohlson model. The main difference between the results is that the parameter on NI/TA becomes negative and significant in Zmijewski's model. This is the result we would, *a priori*, expect. The higher the income, the lower the probability of default. (Note that although this variable has a positive sign in Ohlosn model, it is insignificant.) CA/CL has a positive and significant parameter. The magnitude of the parameter, 0.069, is also very close to that estimated by the Ohlson model, 0.062. This confirms that the result is not due to an interaction with other variables that are highly correlated, for example WC/TA. Rather, for our sample, it suggests that liquid assets do not play an important role in determining firm defaults.

According to the Shumway model [Shu], which suggests a prominent role for market variables, firms with low idiosyncratic risk, measured by Sigma, good past stock performance relative to the market, measured by Diff-Ret, and large market capitalization relative to the market, measured by Rel-Size, should have lower default probabilities. We find that Sigma has a positive and significant coefficient with a t-statistic of 3.769, and Diff-Ret has a negative estimated parameter, as should be expected, that is also statistically significant at the 99% confidence level (t-statistic -5.786). Rel-Size appears unrelated to the probability of default, as its t-statistic is -1.395.

Now, we turn to the parameter estimates of our two duration models. First, *Private* [Pvt] confirms the findings of the other models: a firm with high operating income, high income growth, and low leverage ratio is less likely to default. Not predicted by the other models, Age is also related to financial distress.

Our primary model, *Duration* [Dur], uses both accounting and market variables. We find that TL/TA is positively related to the default probability with a t-statistic of 10.236, as is Sigma, which has a t-statistic of 3.791. Other variables that are negatively related to default risk and are statistically significant at the 1% significance level are CHIN, Diff-Ret, Q, and Age. The age effect is clear: young firms are more likely to default. This is also suggested by the *Private* model. Mature firms have likely established a rapport with credit institutes and private investors that would, in part, alleviate asymmetric information problems because an extended period of scrutiny would permit a better understanding of the economic viability of the firm. In the case of a liquidity crunch, an older firm could rely on such a relationship to obtain additional lines of credit or favorable grace periods, thus avoiding a corporate default event.[10] On the other hand, young firms have less time to solidify a relationship with creditors and private investors,

[10]Diamond (1989), for example, analyzes the incentive effect of reputation on mitigating conflicts of interest between borrowers and lenders. In his model, in the presence of severe adverse selection, reputation will play a role only after the firm establishes a long history of financial solidity (repayment record), given by a lack of defaults. After the firm has established a good reputation, the cost of borrowing is reduced.

increasing the likelihood of financial distress during a credit crunch. From an empirical point of view, one could however argue that age is just a proxy for size; at the univariate level the two variables have a positive correlation coefficient of 0.487. When we leave out Age, however, the parameter on Rel-Size remains insignificant. Additionally, if we include an interaction variable between Age and Rel-Size, the parameter on the interaction variable is negative and statistically significant (t-statistic = -4.9), and does not alter the significance of the Age parameter. This indicates that a firm's age plays a role independent of size in predicting the likelihood of default. The significance of the Age effect is important because it gives validation to the choice of a duration model for the purpose of computing the default probability.

We also investigate the economic significance of each of the variables from the above models. Marginal effects quantify how much the probability of default changes, *ceteris paribus*, when one of the variables changes by some discrete amount. For both the logit and duration models, the partial derivatives of the default probability, which enter in the marginal effects formula, depend on the value of the variables.[11] A natural choice is then to evaluate the effect at the mean value, and to consider a change in the level equal to either some percentage of the mean or some multiple of its standard deviation. In the top part of Table III, estimates of marginal effects are based on a change equal to one standard deviation. As can be seen from Table I, some of the variables are quite skewed and, as a result, the standard deviations can be affected by extreme outliers and may not be representative statistics. The standard deviations used when we compute the marginal effects are hence obtained by truncating the variable distributions at the 1^{st} and 99^{th} percentiles. The second column of the table, Δx, reports the value of the standard deviation, while the third column, $\Delta x\%$, gives an idea of the percentage change with respect to the mean of the variable.

To interpret the magnitudes of the following results, we need to keep in mind that the probability of default for the average firm is about 0.30% and that all of the models classify as in default any firm with a probability higher than 1.5%. For some variables, the marginal effects are similar in magnitude across models. The variables that have the largest impact on the default probability are TA/TL, CHIN, and Q. Further, an increase in TL/TA of one standard deviation, for example, corresponds to an increase in leverage of 17%. Such a movement produces an increase in the distress risk measure for the average firm of about 0.21% to 0.35%. An increase in income growth of 57% is associated with a reduction in exposure to default by 0.14% to 0.19%. An increase in Q of 0.83 decreases risk by 0.13% basis points. The importance of

[11] The marginal effect is given by the partial derivative of the probability of default with respect to a variable multiplied by the variable change. The logit and duration marginals are given by $\Delta\hat{P}(y=1|x) = \hat{\beta}\frac{\exp(\bar{x}\hat{\beta})}{[1+\exp(\bar{x}\hat{\beta})]^2}\Delta x$ and $\Delta\hat{P}(y=1|x) = \frac{\partial h}{\partial x}\exp[-h(\bar{x},\hat{\beta})]\Delta x$, respectively, where $h(x,\beta)$ is the hazard rate.

the other variables is more model-specific. For example, in the Shumway model, an increase of 47% in Diff-Ret produces a 0.16% reduction in default probability. In our Duration model, the reduction is only 0.07%. Examining the marginal effects in both duration models, we note that Age has an economically significant effect. For the average firm, an increase in age of 11.8 years corresponds to a 0.08% decreased risk of default.

B. The Accuracy

Though the parameter estimates are interesting, it is more economically relevant to determine which model best forecasts defaults. We compare each model's performance in two ways: we look at the classification ability of the model for both *Default* and *Non-default* firms (total performance) and we also compute measures that specifically focus on the model's precision relating only to the *Default* group. Results of this analysis are reported in Table IV.

In analyzing total performance we first find a value for the probability of default that optimally separates in-sample defaulted firms from the others.[12] Then we apply this cut-off point to the out-of-sample validation group: firms with a predicted probability of default higher than the cutoff are classified as *Default*, and firms with a predicted probability lower than this critical value are classified as *Non-Default*. We proceed by by computing the percentage of firms that are correctly assigned to their group. We then organize these results into a matrix, in which we note the percentage of correct and incorrect classifications of *Default* and *Non-Default* firms. Using this information, following Altman, Haldeman, and Narayanan (1977), we construct an indicator of how costly incorrect classifications are. We call this indicator the Error Classification Measure (ECM):

$$ECM = p_I P(II|I)c_I + p_{II} P(I|II)c_{II}, \tag{4}$$

where c_I stands for the cost of classifying a non-default firm as a default firm (type I error), and c_{II} represents the cost of classifying a default firm as a non-default firm (type II error). c_I represents the opportunity cost attached to the decision of not investing in a good project, and c_{II} represents the loss associated with a bad investment. We identify c_I as the spread between corporate and treasury bonds and c_{II} as the percentage loss upon default. In their studies on credit spreads, Elton, Gruber, Agrawal, and Mann (2001) report spreads between BBB corporate and government bond yields that vary from 1.15% to 1.5%, depending on maturity and industrial

[12]For discriminant analysis the optimal cutoff point can be proved to be the score that lies at exactly the same distance from the mean scores of the two groups. Since this is a pretty simple and intuitive rule, we determine the cutoff point for logit and duration model in the same way.

sectors. We choose $c_I = 1\%$ as a conservative estimate. Franks and Torous (1989) report an average recovery rate of approximately 60%, so we fix $c_{II} = 40\%$. As for the other parameters, p_I and p_{II} are the unconditional probabilities of the two groups (group I is the healthy firm group; group II the default group). For estimates, we use the empirical frequencies observed year by year. $P(II|I)$ is the probability that the model classifies a firm which is really group I as belonging to group II, and $P(I|II)$ is the probability that the model classifies a firm which is really group II as belonging to group I. We obtain estimates for these conditional probabilities from the model's percentage of incorrect classifications. We believe ECM is a good measure because it weighs *both* type I and type II errors, therefore giving an overall indication of a model's performance in terms of an economically sensible measure.

Panel A of Table IV reports the *Total Performance* measures: in the four columns on the left side of the table we present the percentages of out-of-sample correct and incorrect classifications for *Default* and *Non-Default* cases. For example, optimally dividing the z-scores produced by the Alt model, we *ex ante* correctly classify 66.4% of *Non-Default* firms and 59.7% of *Default* firms. Comparing different models, we notice that the Dur model has the highest accuracy, assigning 89.3% of *Non-Default* firms and 67.2% of *Default* firms to their respective groups. Although the Shu model is less accurate than the Dur model by 3.4% in both type I and type II classifications, the differential in performance is more prominent when the Dur model is compared to other models: the correct default prediction is 15.6% higher than the Pvt forecast, 17.2% higher than the Zmi forecast, and 10.7% and 8.5% higher than the Ohl and Alt forecasts, respectively. In the rightmost column we report the ECM, that represents the cost of incorrect classification, measured as the number of basis points per dollar invested. The lower the ECM, the better the model. An overall comparison reveals that our Dur model has the lowest ECM, followed by Shu which has an additional opportunity cost of 4.5 basis points. The magnitude of this difference might seem quite small at first. However, to really understand the cost differential, this number must be related to the type I error cost that we fix at 1%. A "more realistic" higher cost structure, that eventually increases with other bond characteristics such as the maturity, coupon, and rating would produce a greater cost differential. With a cost of 1%, the reported ECMs imply that implementing a capital allocation rule using the Dur duration model as opposed to the Shu model would allow savings equal to 5% of the part of the accrued interest that is attributed to the credit spread. In this light, the stronger performance of our Dur model is noteworthy.

As an alternative measure of performance, we investigate how accurate the models are in predicting solely default events. We sort the risk scores into deciles and then count the number of defaults in each decile. The accuracy of a model is determined by the percentage of defaults that are classified in the highest deciles. Finally, a more synthetic measure of model accuracy is provided by Sobehart and Stein (2000), and is called the Accuracy Ratio (AR). To construct the AR, we count how many firms actually default within the $\alpha\%$ of firms with the highest probability of default, $0 \leq \alpha \leq 1$. We then plot this cumulative percentage of correct classifications on the unit interval, obtaining a curve that takes the value 0 at $\alpha = 0$ and the value 1 at $\alpha = 1$. This curve is called the Cumulative Accuracy Profile (CAP). A theoretically *perfect model* would predict all defaults in the first $\hat{\alpha}\%$ of firms, where $\hat{\alpha}$ is equal to the frequency of defaults in the sample. The plot of the perfect model would be a straight line for all $\alpha < \hat{\alpha}$ and would be equal to 1 for $\alpha \geq \hat{\alpha}$. A random model, on the other hand, would assign an equal number of defaults to every α; once plotted, it would generate a 45 degree line. AR is defined as the ratio of the integral above the 45 degree line of the model's CAP to the same integral for the perfect model.

For each model, Panel B of Table IV reports the decile percentage of correct classification and the ARs. On the top part of the panel we show the percentage of correct classifications. For example, the first row indicates the percentage of out-of-sample defaults we find in the first decile of firms with the highest probability forecast. The most accurate model is the one with the highest percentage. As can be seen, the Dur duration model performs the best, assigning 62.1% of the cases to the highest probability decile; Shu and Pvt achieve 50% and Ohl, Zmi, and Alt achieve 46.8%, 35%, and 19.4%, respectively.

The last row of Panel B reports *Accuracy Ratios*. AR is a unit-free measure. We can think about it as an indication of how close each of the discussed models gets to the perfect model (the one with maximum theoretical efficiency). An AR of 0.8 suggests that the model's accuracy is 80% of the maximum.[13] An overall comparison reveals that the Dur duration model achieves the highest AR (equal to 0.698), closely followed by Shu (0.686). The ARs of the other models are smaller: Pvt and Ohl have ARs of approximately 0.600, while Zmi and Alt have ARs equal to 0.518 and 0.359, respectively.

This first set of results unambiguously suggests that the model we propose in this paper (the Dur model) is a better statistical model then those found in previous literature. We attribute this result not only to a better choice of conditioning variables, but also to a better econometric

[13] AR has one undesirable property: it depends on the entire Cumulative Accuracy Profile. In other words, it depends on the accuracy of the model in percentiles that are far away from what is economically sensible. If a model has a high percentage of default in the first decile, it is probably a good model. But the fact that it reaches a high cumulative percentage by the fifth decile may not signal too much about the model's performance when applied to real-life cases. For example, Shu and Pvt have the same percentage of default firms in the first decile, and a small difference in the second; Shu, however, has a higher AR than Pvt. Though Dur has a much higher percentage in the first decile when compared to Shu, its AR has is only slightly better. Nonetheless AR is a good indicative measure.

specification. Without imposing computational complexity, our duration model allows a correction of sample selection bias, and time varying regressors, while maintaining maximum flexibility in the specification of the hazard function.

To ensure that our conclusions are representative, as a robustness check, we repeat our estimation procedure many times, each time moving the validation sample ahead one year. We start by estimating the models with the observations belonging to the years before 1982 as the estimation sample and those following 1982 (1982 included) as the validation group. We then use observations belonging to the years before 1983 to estimate the models, and those after 1983 to form out-of-sample predictions, and so on. We report out-of-sample ECMs and ARs in Table V (left and right sides, respectively). Our conclusion that the Cpl duration model has the lowest ECM is confirmed in almost every year between 1982 and 1999. In 1983 and 1984, there is not a statistically significant difference in performance between Dur and Ohl. Our duration model also has a better AR in nearly every year. The exceptions of 1983 and 1999 favor Shu model, but again the difference is modest. In all other years the Dur model has a better ratio, on the order of 1-4%, with an average of 2.4%. These results confirm that the model we propose performs better.

IV. Pricing Default

A. Stock's Default Probability and FF Factors

In this section we investigate how our measure of financial distress, estimated by the Dur duration model, relates to stock prices. We focus on the Dur model because of its higher accuracy in predicting both *Default* and *Non-Default* firms. Forecasting precision is imperative as we are going to use the default measure (the predicted probability of default) to examine the relation between distress risk and the FF factors, SMB and HML. The more informative the measure, the easier and more complete our understanding of what the FF factors represent. An inaccurate default measure could lead us to the incorrect conclusions. For example, consider a model that is based on some accounting and market variables that contain information about past performance. Moreover, suppose that the model is poor in that it produces a default measure that is only related to past performance and does not have any value in terms of predicting default. If we were to rank firms based on this model's estimates, we would likely assign a very low default probability to firms with a very good past performance and very high default probability to firms with a very poor past performance. If, as expected, this partition of firms is completely unrelated to the true probability of default, we would expect the stock returns of the first group not to be smaller than the stock returns of the second group of firms. Since it is has been reported that there exists a "momentum effect" in stock prices (Jegadeesh and Titman (1993)) we would likely observe slightly higher returns corresponding to the group of firms that performed well in the past and slightly lower returns in the group of firms that performed poorly in the past. We would then conclude that there is no risk-based relation between stock returns and financial distress, because, if there were, we would have observed it in the returns of these extreme portfolios.

Similar to Dichev (1998), Griffin and Lemmon (2002), and Vassalou and Xing (2004) we form portfolios into deciles by sorting according to the predicted default probability. To account for lags in the availability of financial statement information, we form portfolios in June of every year. We keep the portfolio stock composition for 1 year (until the following May) while monthly rebalancing the portfolio weights. The deciles are obtained by sorting the out-of-sample estimates of the probability of default. These probabilities are determined from accounting and market information from the end of the previous year. Moreover, to avoid relying on non-available information, we use the parameters based on the model estimation of two years before. In this way, our portfolio formation resembles a real time implementable trading strategy.

Contrary to the papers cited above, we use a different default probability model and we focus on testing the hypothesis that the FF factors are related to a (in particular, *our*) default risk measure, rather than trying to explain directly the size and book-to-market effects. In particular, we try to establish a connection between returns and the probability of default, and between the probability of default and factor loadings. We anticipate that expected returns and default risk are related. A high probability of default should cause a stock to have high loadings on the HML and SMB factors, thus delivering high expected returns which compensate the investor for holding default risk.

In Table VI we report returns on the 10 portfolios for each model, before and after risk-adjusting by a 4-factor model: the Fama and French (1993) 3-factor model plus a momentum factor.[14] We find that, in our sample, the portfolios obtained sorting on the risk measures implied by Altman, Ohlson, Zmijewski and Shumway models exhibit value-weighted returns which do not increase with the likelihood of financial distress. In particular, the portfolio corresponding to the lowest probability of default outperforms the other portfolios. The risk adjusted returns are also statistically significant and quite substantial: they range from 51 to 66 basis points per month. From the table, we notice that when we use our more accurate duration model,

[14] The distress factors as well as the momentum factor time series are all publicly available on Kenneth French's website. A momentum factor is added to the three factors model because the default measure loads on past economic performance as well as past stock returns. There could hence be a momentum component to the portfolio returns.

average returns appear be higher the higher the default measure. Moreover, the mispricing in the model's α seems to disappear. Our model's result is more economically intuitive, since investors should be compensated for holding stock of risky firms. We attribute this result to a better ability of the duration model to measure default risk at both extremes of the empirical distribution of estimated probability of default.

The top part of Table VII reports the average characteristics of the decile portfolios when we use the probability of default obtained from our Cpl model. Again, by-and-large, the portfolios exhibit returns increasing in the risk dimension. On the other hand, the return standard deviations display a U-shaped relation. The standard deviations of the extreme portfolios are the highest. More striking patterns are shown by the average size, book-to-market and volume of the firms in each decile. The average size of the stocks in the first decile approximates 3 billion dollars, with an average monthly volume of 464 million shares. The average size of the stocks in the last (highest probability of default) decile is 358 million dollars, smaller by an order of magnitude, with a volume of 33 million shares. Size and volume are almost monotonically decreasing along the default dimension.The average book-to-market is increasing in the default measure. Firms with high default probabilities have higher book-to-market values, 0.849 on average, while firms with low default probabilities have relatively low average book-to-market value, 0.578.[15]

For each decile portfolio, we also run a time-series regression of the value-weighted returns on a four-factor model. The bottom part of Table VII reports the estimates of the factor loadings along with robust Newey and West (1987) t-statistics. We note that β_{mkt} is highest at the extreme deciles and decreases moving from the extreme to the center: the 6^{th} decile portfolio loads on the market with the lowest β_{mkt}, 0.899. The very risky portfolios in deciles 9 and 10 have the highest market betas, that are well above 1 (1.081 and 1.257, respectively).

If SMB and HML are related to financial distress, we would expect the portfolio loadings to increase along the dimension of default risk. Indeed, this is what we find. The result is much stronger for HML than SMB. The β_{HML}s monotonically increase from -0.623 in the 1^{st} decile to 0.737 in the 10^{th}. Only two parameters are not statistically significant at the 95 or 99 confidence levels (though one of them is significant at 10%). The magnitude of the estimates is in line with that reported by Fama and French (1993) wherein high and low book-to-market portfolios have respective loadings approximating 0.70 and -0.40. The effect is less dramatic when loadings on SMB are considered. The β_{SMB} still increases from decile one to decile ten, but the relation is

[15]The book-to-market of the average firm in the first decile might at first appear fairly large. However, when we compute the book-to-market of the portfolio, weighing the book-to-market of the single firms by their size, as is done in computing returns, we obtain a more reasonable 0.318 (unreported).

less clear within the middle deciles.

We also investigate how individual stocks (as opposed to portfolios) relate to default risk. If the factors are pricing systematic distress risk, it should be that individual firm loadings on the factors are related to their estimated probability of default. Every month we estimate individual stock loadings for the 3-factor model using a 60-month window and requiring that any stock has at least 36 monthly observations in every window. In June of every year, we sort the stocks into decile portfolios based on the probability of default; hence, for each default portfolio, we compute the average loading for the stock in that portfolio at the portfolio formation date. We report results at the bottom of Table VII. We find strong evidence that the individual loadings on the HML factor are related to default risk. As we can see from the table, the loadings on the sorting date are monotonically increasing. The average firm in the low probability of default decile has a negative loading on HML and that in the high probability of default has a large positive loading. The evidence on the SMB loadings is less conclusive. The average SMB loading is increasing in deciles 6 to 10, but is decreasing in deciles 1 to 6.

A more rigorous test of the relation between default probability and factor loadings at the individual stock level involves a two-stage Fama and MacBeth (1973) procedure. In the first stage, for each stock, in June of each year, we run a time-series regression using a 5-year window of monthly data to obtain estimates of the stock loadings on the factors. In the second stage, in June of each year, we run a cross-sectional regression of a logistic transformation of the probability of default, $\bar{P}_{it} = \log\left(\frac{P_{it}}{1-P_{it}}\right)$, on the individual stock factor loadings, $\hat{\beta}_{SMB,it}$ and $\hat{\beta}_{HML,it}$, and a set of some control variables. Specifically:

$$\bar{P}_{it} = \lambda_{0,t} + \lambda_{SMB,t}\ \hat{\beta}_{SMB,it} + \lambda_{HML,t}\ \hat{\beta}_{HML,it} + \lambda_{x,t}\ x_{it} + \epsilon_{it}$$

Then we compute time-series average and robust standard errors of the cross-sectional estimates[16]

$$\hat{\lambda} = \frac{1}{T}\sum_{t=1}^{T}\hat{\lambda}_t, \qquad SE(\hat{\lambda}) = \frac{1}{T}\sum_{j=-\infty}^{\infty} cov_T\left(\hat{\lambda}_t, \hat{\lambda}_{t-j}\right)$$

We test the null hypothesis that there is no relation between the factor loadings and the probability of default against the alternative that there is a positive relation for λ_{SMB}, λ_{HML}:

$$H_0 : \lambda = 0 \text{ against } H_1 : \lambda > 0$$

[16]A detailed description of how to estimate robust standard errors that take into account autocorrelation in the estimates can be found in Cochrane, John H. (2001), section 11.7.

Table VIII reports results. The base case regression, column (1), relates the SMB and HML loadings to the probability of default and highlights a positive and statistically significant relation. Both $\hat{\lambda}_{SMB}$ and $\hat{\lambda}_{HML}$ are significantly positive with respective t-statistics of 2.39 and 7.76. Column (2) gives the estimates of a model which uses only size and book-to-market as opposed to the factor loadings. Both variables have a statistically significant estimated coefficients. Given the evidence reported in the top of Table VII, this is exactly what we would expect. On the other hand, market beta does not appear related. The estimated coefficient in column (3) is positive, but not statistically significant. We also re-estimate model (1) including size and book-to-market as control variables. If the FF factor loadings are, in fact, related to the default probability we expect that the parameter estimates remain essentially unchanged, thus retaining statistical significance. We find that $\hat{\lambda}_{HML}$ maintains its sign and significance (t-statistic = 7.28). On the other hand $\hat{\lambda}_{SMB}$ appears with a negative sign, suggesting that the SMB factor may not be capturing any distress risk.

B. Aggregate Default Risk and HML

The results reported thus far suggest that the HML factor is cross-sectionally related to a firm measure of financial distress. However this is not enough to confirm the interpretation of HML as a distress factor. In particular, the fact that firms with higher probabilities of default have higher loadings on the factor does not justify a premium for these stocks. Firm distress is just another idiosyncratic risk, and as such, it can be diversified away. On the other hand, if the stock performance of distressed firms worsens when the economy is in a state of high distress risk, the investors, who hold high-default-probability stocks, are exposed to systematic default risk. As a compensation for holding this risk, they require a distress premium. This would imply that the HML factor covaries with measures of aggregate financial distress.

Lakonishok, Shleifer, and Visnhy (1994) notice that in order to justify a risk-based premium the FF factors returns should be related to aggregate risk news. At times when the marginal utility of wealth is high the return of the factors should be particularly low. If the factors proxy for default risk, when aggregate default risk increases stocks that are more likely to default (high book-to-market) should substantially underperforme stocks that are not so likely to default (low book-to-market). Since aggregate default is a non-diversifiable risk, an investor could hence require a premium for holding financially distressed firms. This imply that the average return of the FF factors should be positive, but characterized by large negative realizations in periods distinguished by an increase in aggregate distress risk.

In Table IX we report correlation coefficients between year-to-year changes of the aggregate default probability (ΔDEF), and the returns on SMB, and HML for the years 1961-2002. We compute the default probabilities for the years that are outside our original sample using the average of the 1983-2000 time-series of the hazard rate estimated paramters. The aggregate default probability is obtained by taking a simple value-weighted average of the firm probabilities. Moreover to account for the lag in availability of accounting information we compute returns on the factors from June to May of each year. If SMB and HML were distress factors we would expect ΔDEF to be negatively related to SMB and HML. Similarly to Vassalou and Xing (2004) we find that ΔDEF is positively related to SMB (18% and not significant), and negatively and significantly correlated with HML (-35%).

We investigate the economic significance of this result in a non-parametric way by looking at the average return of the factors in years characterized by an increase in the aggregate default risk measure. In particular we look at the subsets corresponding to an increase bigger than respectively the average, the 75^{th} percentile ($q(75)$), and the 90^{th} percentile ($q(90)$) of the distribution of ΔDEF. The hypothetical relation between distress risk and the FF factors leads us to expect that the higher ΔDEF is, the lower the contemporaneous return of the factors. This hypothesis cannot be rejected for HML. We observe that in the four years characterized by the highest change in the aggregate default measure, $\Delta DEF > q(90)$, the return on HML averages -4.5% per year while in the remaining 37 years it has an average return of 6.7%. In the subset identifyed by the biggest decreases, $\Delta DEF < q(10)$, the mean return is instead 12.4%. On the contrary the positive correlation with ΔDEF and the return pattern of SMB does not accord with the interpretation of the factor as a distress factor.

As a robustness check we repeat the analysis for the two subsamples 1961-1981 and 1982-2002. We find that the relation between ΔDEF and HML is negative in both subsamples, though it is significant only in the latter time-period. The negative performance is not quite present in the first subsample. This result could however be due to the look-ahead bias in the COMPUSTAT dataset reported by Kothari, Shanken, and Sloan (1992). In the 1982-2002 period the result is instead stronger. In the five years associated with the largest increases in aggregate default the average return is around -7%. In the remaining 15 years HML returns an average 8.3% per year. Again we find no evidence in support of SMB as a distress factor in any of the two subsamples.

This set findings supports the hypothesis that HML could be related to an idyosincratic source of aggregate default risk. The HML (value) premium comes from the fact that in years in which the economy is characterized by a greater default risk firms that are more likely to default, high book-to-market, tend to underperforme firms with low probability of default. In these years a strategy that buys the first and sell the last, like HML, would have a negative return. Since investors cannot diversify this risk, they require a positive premium.

As a further robustness check we investigate the nature of the correlation between HML and news to the aggregate distress risk analyzing data at higher frequency. A regression framework, made possible by the increased number of observations, and the use of other conditioning variables allow us a better investigation of the hypothesized negative relation. We expand the original sample to the period between 1969 and 2002 and we construct the financial ratios we need using the quarterly financial statements reported by COMPUSTAT.[17] For each quarter the aggregate measure, henceforth DEF, is given by the value-weighted average of the firm's distress probabilities. In order to account for all possible model misspecifications we run several versions of the basic model

$$HML_t = \beta_0 + \beta_1 \Delta DEF_t + \beta_2 x_t + \mu_t$$

wherein x_t is a vector of control variable that includes: change in credit spreads, the market return, the return on SMB, a recession dummy, and an interaction variable between the recession dummy and ΔDEF_t. Table X presents estimated coefficients and robust t-statistics. We obtain the parameter estimates and t-statistics of the models in columns (1) and (3) to (8) using OLS and Newey and West (1987) robust standard errors, using 2 leads and 2 lags. In column (2) we report the results of the estimate of the basic model using Hansen (1982) efficient GMM, instrumenting ΔDEF_t with the change in the aggregate Tobin's Q and with ΔDEF_{t-1}.

We find a statistically significant negative relation between ΔDEF_t and HML in the basic models (1) and (2). The change in credit spreads is negatively, but not significantly related to HML, column (3). Controlling for market risk, column (5), strengthens the size and significance of both parameters (ΔDEF_t and $\Delta Spread_t$). Although not completely significant we find evidence of a recession effect, column (6). The conditional mean of HML is indeed higher as well as the negative correlation between ΔDEF_t and HML is stronger in recession quarters. Finally, the basic results are confirmed in the two sub-samples: 1969-1985 and 1986-2002.

[17] We cannot use this expanded sample to estimate the predicting model because we only observe default events for the period 1979-2000, and for the first half of the sample we know only the year corresponding to the events so that it would be impossible to use quarterly frequency.

V. Conclusions

In this paper we study how corporate bond defaults can be predicted using financial ratios and how the predicted probability of default relates to the cross-section of stock returns. Using public data, we construct a sample of non-financial firms that have been traded in one of the three US stock markets between 1979 and 2000 and have some public debt. Default events have been hand-collected using primarily Moody's database and publicly available press-news.

We propose and apply a simple piece-wise constant hazard model, which is not only easy to estimate but flexible in the choice of the base hazard specification. Results indicate that, in the sample we consider, our duration model outperforms some of the most cited bankruptcy models. Our model, when compared to the Altman (1968), Ohlson (1980), Zmijeski (1984), and Shumway (2001) models, is more accurate in correctly classifying both *Default* and *Non-Default* firms.

Further, we also analyze the relation between default risk (as predicted by our model) and the Fama and French (1993) distress factors, HML and SMB. In other words, we try to establish a connection between returns and the probability of default, and between the probability of default and factor loadings. We expect that returns and default risk are related. A high probability of default should cause a stock to have high loadings on the HML and SMB factors, thus delivering high expected returns which compensate the investor for holding default risk. We provide concrete evidence that both portfolio and individual stock factor loadings are related to the probability estimates that are provided by our model. Moreover, if the FF factors represent a systematic distress risk, we should observe that a high realization of the distress factors is associated with news about aggregate default risk. Indeed, we find that change in our aggregate measure of default risk are significantly negatively correlated with HML realizations.

The main contribution of this paper is to demonstrate the link between expected stock returns and distress risk, reconciling the interpretation of HML as a distress factor. An interesting future development of the current research would be to look at the role of the aggregate measure of financial distress in directly pricing the cross-section of average stock returns. Researchers have recently studied conditional versions of the static CAPM and the *Consumption* CAPM. For example, Lettau and Ludvigson (2001) use the deviation from the long-run mean of the log consumption-wealth ratio as a conditioning variable. Along these lines, it would be interesting to investigate the role of our measure of aggregate distress risk as a conditioning variable in a conditional asset pricing model.

Appendix

A. Variables

Variable	Description
WC/TA	WC/TA is a measure of the net liquid assets. It is relevant because, when facing operating losses, WC tends to be negative, hence indicating that the firm is approaching a state of financial insolvency.
RE/TA	RE/TA is considered a good measure of the accumulated value of the firm. Retained earnings reflect past performance and represent the intent of the management to make part of the profits available for any use, including financial obligations the firm may have to meet. Of course, young firms have had less opportunity to generate cumulative profits, making them more likely to default in bad economic times.
EBIT/TA	EBIT/TA is considered a good measure of the true profitability of the business regardless of tax and leverage considerations.
MV/TL	is a market measure of the firm capital structure.
S/TA	S/TA illustrates the sales generating ability of the assets in place; it is important because sales not only represent the main source of cash flow, but also offer a good measure of management's ability to compete with other firms.
log(TA)	log(TA) may affect the likelihood of default as well. Larger firms have more possibilities to substitute assets, liquidate existing operations in order to generate cash, back additional line of credits with real guarantees.
TL/TA	TL/TA is measured as the percentage of total liability to total assets in book value; it gives an idea of the financial structure of a firm. In principal we would be like to know the exact composition if the financial structure of each firm. It is not difficult to conjecture that if creditors had some stake of the firm's equity, for example, it could be in their interest not to put the firm in default. Information about the financial structure are not available at this level of aggregation, though, so we relay on the leverage ratio.
CA/CL	CA/CL is another key source of information, as current assets reflect the main source of liquidity (cash and short term investments which can be easily liquidated). Thus, they represent the ability of the firm to make interest and principal payments. CL represents the debt that is going to be liquidated in the next year, and thus provides a measure of short-term financial obligations.
INTWO	INTWO is a dummy variable that takes value of one id NI was negative for the last two years, and zero otherwise.
CHIN	CHIN is computed as $(NI_t - NI_{t-1})/(\|NI_t\| + \|NI_{t-1}\|)$, and is intended to measure change in net income.

Variable	Description
Diff-Ret	Diff-Ret is measured as the buy and hold return of the firm's stock over the past year minus the weighted average CRSP portfolio return over the same period.
Sigma	Time t Sigma is calculated by first regressing the monthly stock returns of every firm in year $t-1$ over the weighted average CRSP portfolio returns over the same period, and then by taking the standard deviation of the regression residuals.
Rel-Size	Rel-Size is constructed as the logarithm of the ratio of the market capitalization of a firm at the end of the year to the total market capitalization.
AGE	AGE is defined as the number of years a firm has been traded on one of the three major US markets. It is an important variable because more mature firms should have well established credit relations, that make defaults less likely to happen. For opposite reasons young firms are at a higher risk of financial distress.
COV	COV provides a crucial information because interest expenses are the trigger of corporate defaults; when the firm does not have enough cash to repay interest, unless other sources of credit become available, default is unavoidable. We compute COV as the difference between operating income and interest expenses divided by the absolute value of operating income. In this way, the ratio is monotonically increasing in the ability of the firm to cover interest expenses. If the firm does not have any other liquid asset, other than the cash generated by the main operations, there could be a potential endogeneity problem, due to the fact that default and coverage ratio would be perfectly correlated and contemporaneously determined.
Q	Q is widely used in the economic literature as an indicator of growth possibilities. When growth possibilities are abundant, it should be easier to raise additional credit, and less likely that the firm enters a state of financial distress. Q is computed as the ratio of the sum of MV and TL to TA, where TL and TA are at book values.

B. Likelihood Specification

We estimate the parameters of our model using a simple maximum likelihood approach. The specification of the likelihood function depends on both the sampling scheme we adopt and the characteristics of the data. For example, the presence of incomplete durations implies the use of right censoring. As mentioned before, there are two different sampling schemes: *flow sampling*, wherein firms are observed as they flow in the sample after a particular point in time, and *stock sampling*, wherein all firms present at a particular point in time enter the sample and are followed until their duration is complete. In our case, we begin observing firms in 1979 and require that their stocks are traded in one of the three major US markets. If we used flow sampling, our sample would include all firms that begin being traded after 1979. If we used stock sampling only the firms that were in the market in 1979 or those that enter afterwards would compose our sample. The use of those sampling scheme depends on the nature of the problem at study. When we observe a sample in which duration is not extremely long, and durations are complete (by which we mean that firms have completed their transition from one state to the other), *flow sampling* seems to be a better choice. When durations are long and firms tend to outlast the observation period, adopting a *flow sampling* results in not considering a significant portion of the firms.

We derive our likelihood function beginning from the simpler flow sampling case, in which we would have

$$\ell = \log \prod_{i=1}^{N} [f_i(y \mid \beta, x)]^{d_i} [S_i(y \mid \beta, x)]^{1-d_i} = \log \prod_{i=1}^{N} [\theta_i(y \mid \beta, x)]^{d_i} S_i(y \mid \beta, x) \tag{5}$$

where $y = \min(t, c)$, t is the duration, c is the censoring time (the number of units of time after which we stop observing the sample), and d is the censoring indicator, which takes the value 1 when the i firm is not censored, hence the duration is complete, and zero otherwise.

Though it has the advantage of considering more firms, *stock sampling* introduces a sample selection problem due to the fact that firms with a longer duration are more likely to be sampled. A natural solution is to use the total duration of those firms that were already in the state before we start our sampling. If we call e the elapsed time between the firm entering the state and the beginning of the sample period[18], we can condition the survival function on the fact that we already know the firm has survived for at least e periods:

$$P(y > t \mid y > e) = \frac{P(y > t)}{P(y > e)}$$

[18]For example, consider a with CRSP and COMPUSTAT data between 1970 and 1992. Though its duration (measured as the number of years the stock is present in CRSP) is 22 years, it is in our sample only between 1979 and 1992. e is then equal to 9 years, corresponding to the period between 1970 and 1979.

where the equality holds because $t > e$. We can now incorporate this correction and write the stock sampling likelihood function as follows:

$$\ell = \log \prod_{i=1}^{N} [f_i(y \mid \beta, x)]^{d_i} \left[\frac{S_i(y \mid \beta, x)}{S_i(e \mid \beta, x)} \right]^{1-d_i} \tag{6}$$

The sampling scheme we adopt in this study is a mix of the two: we employ a basic stock sampling and we also include all new entrants after 1979. In other words if $e = 0$, $P(y > e) = 1$, which is enough for consistency of Equation 6.

We need one more adjustment: Equation 6 assumes a continuous duration. Duration, however is discrete and grouped into one-year time intervals. The corrected log-likelihood function is therefore written as:

$$\ell = \log \prod_{i=1}^{N} [S_i(y-1 \mid \beta, x) - S_i(y \mid \beta, x)]^{d_i} \left[\frac{S_i(y \mid \beta, x)}{S_i(e \mid \beta, x)} \right]^{1-d_i} \tag{7}$$

With time-varying regressors and stock sampling, the log likelihood function, Equation 7, becomes

$$\ell = \log \prod_{i=1}^{N} [S_i(y_i - 1 \mid \beta, x(1), \ldots, x(t_i - 1)) - S_i(y_i \mid \beta, x(1), \ldots, x(t_i))]^{d_i} \left[\frac{S_i(y_i \mid \beta, x(1), \ldots, x(t_i))}{S_i(y_i \mid \beta, x(1), \ldots, x(e_i))} \right]^{1-d_i}$$

where $y_i = \min(t_i, c_i)$ and y_i, t_i and e_i are different among firms.

References

Altman, Edward I., 1968, Financial ratios, discriminant analysis and the prediction of corporate bankruptcy, *Journal of Finance* 23, 589–609.

Altman, Edward I., Robert G. Haldeman, and Paul Narayanan, 1977, Zeta analysis: a new model to identify bankruptcy risk of corporations, *Journal of Banking and Finance* 1, 29–54.

Beaver, William H., 1966, Financial ratios as predictors of failure, *Journal of Accounting Research* 4, 71–111.

Begley, John, Jin Ming, and Susan Watts, 1996, Bankruptcy classification errors in the 1980s: an empirical analysis of Altman's and Ohlson's models, *Review of Accounting Studies* pp. 267–284.

Brennan, Michael J., Tarun Chordia, and Avanidhar Subrahmanyam, 1998, Alternative factor specifications, security charachteristics, and the cross-section of expected returns, *Journal of Financial Economics* 49, 345–373.

Chan, Louis K., Nai-fu Chen, and David Hsieh, 1985, An exploratory investigation of the firm size effect, *Journal of Financial Economics* 14, 451–471.

Chen, Nai-fu, Richard Roll, and Stephen A. Ross, 1986, Economic forces and the stock market, *Journal of Business* 59, 383–403.

Cochrane, John H., 2001, *Asset Pricing*. (Princeton University Press Princeton, New Jersey).

Cooper, Michael, Huseyin Gulen, and Maria Vassalou, 2001, Investing in size and book-to-market portfolios using information about the macroeconomy: some new trading rules, Working Paper.

Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam, 2001, Overconfidence, arbitrage, and equilibrium asset pricing, *Journal of Finance* 56, 921–965.

Daniel, Kent, and Sheridan Titman, 1997, Evidence on the charatceristics of cross sectional variation in stock returns, *Journal of Finance* 52, 1–33.

Davis, James L., Eugene F. Fama, and Kenneth R. French, 2000, Characteristics, covariances, abd average returns: 1929 to 1997, *Journal of Finance* 55, 389–406.

Diamond, Douglas W., 1989, Reputation acquisition in debt markets, *Journal of Political Economy* 97, 828–862.

Dichev, Ilia, 1998, Is the risk of bankrupt a systematic risk?, *Journal of Finance* 53, 1131–1147.

Elton, Edwin J., Martin J. Gruber, Deepak Agrawal, and Christopher Mann, 2001, Explaining the rate spread on corporate bonds, *Journal of Finance* 56, 247–277.

Fama, Eugene F., and Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

Fama, Eugene F., and Kenneth R. French, 1995, Size and book-to-market factors in earnings and returns, *Journal of Finance* 50, 131–155.

Fama, Eugene F., and Kenneth R. French, 1998, Value versus growth: the international evidence, *Journal of Finance* 53, 1975–1999.

Fama, Eugene F., and James D. MacBeth, 1973, Risk, return, and equilibrium: empirical tests, *Journal of Political Economy* 81, 607–636.

Franks, J.R., and Walter Torous, 1989, An empirical investigation of U.S. firms in reorganization, *Journal of Finance* 44, 747–769.

Griffin, John M., and Michael L. Lemmon, 2002, Book-to-market equity, distress risk, and stock returns, *Journal of Finance* 57, 2317–2336.

Hansen, Lars Peter, 1982, Large sample properties of generalized method of moments estimators, *Econometrica* 50, 1029–1054.

Jegadeesh, Narasimhan, and Sheridan Titman, 1993, Returns to buying winners and selling losers: implications for stock market efficiency, *Journal of Finance* 47, 65–91.

Kahl, Matthias, 2002a, Economic distress, financial distress, and dynamic liquidation, *Journal of Finance* 57, 69–77.

Kahl, Matthias, 2002b, Financial distress as a selection mechanism: evidence from the United States, Working paper.

Kothari, S.P., Jay Shanken, and Richard G. Sloan, 1992, Another look at the cross-section of expected stock returns, *Journal of Finance* 50, 185–224.

Lakonishok, Josef, Andrei Shleifer, and Robert W. Visnhy, 1994, Contrarian investment, extrapolation, and risk, *Journal of Finance* 44, 1541–1578.

Lettau, Martin, and Sydney Ludvigson, 2001, Resurrecting the (C)CAPM: a cross-sectional test when risk premia are time-varying, *Journal of Political Economy* 106, 1238–1287.

Liew, Jimmy, and Maria Vassalou, 2000, Can book-to-market, size and momentum be risk factors that predict economic growth?, *Journal of Financial Economics* 57, 221–245.

Lo, Andrew W., 1986, Logit versus discriminant analysis: a specification test and application to corporate bankruptcy, *Journal of Econometrics* 31, 151–178.

McFadden, D., 1976, A comment on discriminant analysis versus logit analysis, *Annals of Economic and Social Measurement* pp. 511–523.

Merton, Robert C., 1974, On the pricing of corporate debt: the risk structure of interest rates, *Journal of Finance* 29, 449–470.

Newey, Whitney K., and Kenneth D. West, 1987, A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix, *Econometrica* 55, 703–708.

Ohlson, James A., 1980, Financial ratios and the probabilstic prediction of bankruptcy, *Journal of Accounting Research* 18, 109–131.

Queen, Maggie, and Richard Roll, 1987, Firm mortality: using market indicators to predict survival, *Financial Analysts Journal* 3, 9–26.

Santomero, A., and J.D. Visno, 1977, Estimating the probability of failure for commercial banks and the banking system, *Journal of Banking and Finance* 1, 185–215.

Shumway, Tyler, 2001, Forecasting bankruptcy more accurately: a simple hazard rate model, *Journal of Business* 74, 101–124.

Sobehart, Jorge R., and Roger M. Stein, 2000, Moody's public firm risk model: a hybrid approach to modeling short term default risk, KMV-Moody's Investors Service.

Sueyoshi, Glenn, 1995, A class of binary response models for grouped duration data, *Journal of Applied Econometrics* 10, 411–431.

Vassalou, Maria, 2003, News related to future GDP growth as a risk factor in equity returns, *Journal of Financial Economics* 68, 47–73.

Vassalou, Maria, and Yuhang Xing, 2004, Default risk in equity returns, *Journal of Finance* 59, 831–868.

Weiss, Lawrence A., and Karen H. Wruck, 1998, Information problems, conflicts of interest, and asset stripping: Chapter 11's failure in the case of eastern Airlines, *Journal of Financial Economics* 48, 55–97.

Zmijeski, Mark E., 1984, Methodoligal issues related to the estimation of financial distress prediction models, *Journal of Accounting Research* 22, 59–82.

Table I
Summary Statistics

This table shows sample statistics of the financial ratios used in the predicting models. The sample period covers 1979-2000. Firms have been divided in two groups: those that do not experience a default event, *Non-Default*, and those that eventually face financial troubles, *Default*. The column labeled Source indicates the literature reference where the variable has been used first. Alt, Ohl, Shu, and New respectively indicate that the variable has been used by Altman (1968), Ohlson (1980), Shumway (2001) or is new to this paper. ** indicates that the difference between the means is statistically different from zero at the 1% confidence level.

Panel A: Non-Defaults

Variable	Source	Mean	Median	Min	Max	Std
WC/TA	Alt	0.213	0.198	-0.899	0.935	0.202
RE/TA	Alt	0.108	0.179	-6.991	1.495	0.533
OIA/TA	Alt	0.116	0.128	-3.430	1.057	0.132
MV/TL	Alt	3.271	1.174	0.006	17557.854	95.686
S/TA	Alt	1.216	1.092	0.000	25.482	0.835
log(TA)	Ohl	5.749	5.709	0.020	12.406	2.114
TL/TA	Ohl	0.571	0.581	0.000	1.000	0.174
CA/CL	Ohl	2.198	1.789	0.000	91.211	2.039
NI/TA	Ohl	0.017	0.041	-4.109	1.513	0.147
fund/TL	Ohl	0.279	0.217	-22.290	666.201	4.675
INTWO	Ohl	0.133	0.000	0.000	1.000	0.339
CHIN	Ohl	0.167	0.105	-1.000	1.000	0.563
AGE	Shu	15.771	12.000	1.000	52.000	12.243
Sigma	Shu	0.109	0.091	0.000	2.742	0.077
Diff-Ret	Shu	0.015	-0.065	-1.280	32.914	0.687
Rel-Size	Shu	-10.037	-10.088	-19.447	-2.868	2.128
Coverage	new	0.171	0.787	-1660.500	1.000	10.289
Q	new	1.611	1.246	0.212	73.426	1.485

Panel B: Defaults

Variable	Source	Mean	Median	Min	Max	Std
WC/TA	Alt	0.205	0.196	-0.843	0.822	0.196
RE/TA	Alt	-0.031**	0.057	-6.576	0.638	0.524
OIA/TA	Alt	0.083**	0.097	-1.091	0.478	0.110
MV/TL	Alt	1.336**	0.633	0.015	59.313	3.228
S/TA	Alt	1.226	1.046	0.006	8.629	0.979
log(TA)	Ohl	5.262**	5.183	0.724	10.539	1.556
TL/TA	Ohl	0.660**	0.664	0.068	0.999	0.168
CA/CL	Ohl	2.113	1.770	0.052	27.868	1.766
NI/TA	Ohl	-0.016**	0.016	-1.343	0.745	0.136
fund/TL	Ohl	0.138**	0.142	-2.666	1.140	0.206
INTWO	Ohl	0.237**	0.000	0.000	1.000	0.426
CHIN	Ohl	0.154	0.107	-1.000	1.000	0.735
AGE	Shu	14.531**	12.000	1.000	51.000	10.563
Sigma	Shu	0.132**	0.118	0.008	0.565	0.070
Diff-Ret	Shu	-0.069**	-0.199	-1.175	5.893	0.696
Rel-Size	Shu	-10.638**	-10.672	-17.701	-4.684	1.615
Coverage	new	-0.748**	0.557	-275.636	0.996	9.157
Q	new	1.288**	1.105	0.468	6.602	0.627

Table II
Parameter Estimates

This table presents parameter estimates and t-statistics. Each model is estimated using the observations between 1979 and 1996 as the estimation sample and the remaining as the out-of-sample validation group. ** and * indicate statistically significance at the 1% and 5% statistical level, respectively.

	Alt	Ohl	Zmi	Shu	Pvt	Dur
intercept		-8.427**	-9.479**	-9.748**	-8.171**	-7.212**
		(-19.303)	(-26.406)	(-20.425)	(-18.134)	(-10.638)
WC/TA	1.805	0.022			0.014	
		(0.058)			(0.036)	
RE/TA	-0.081				0.388*	0.277
					(2.105)	(1.493)
OIA/TA	6.077				-1.908**	-1.720**
					(-3.291)	(-3.567)
MV/TL	0.001					
S/TA	-0.208				-0.015	
					(-0.192)	
log(TA)		-0.074*			-0.032	
		(-2.050)			(-0.780)	
TL/TA		5.199**	6.384**	5.639**	5.240**	5.077**
		(10.858)	(13.871)	(11.795)	(10.550)	(10.236)
CA/CL		0.062*	0.069**		0.057	
		(2.234)	(2.707)		(1.918)	
NI/TA		0.242	-1.060**	-0.256	0.534	
		(0.588)	(-4.840)	(-0.864)	(1.249)	
fund/TL		-0.231			0.001	
		(-1.889)			(0.023)	
INTWO		0.596**			0.478**	0.331
		(3.360)			(2.677)	(1.849)
CHIN		-1.241**			-1.265**	-0.970**
		(-9.929)			(-10.106)	(-7.573)
Sigma				2.482**		2.633**
				(3.769)		(3.791)
Diff-Ret				-1.004**		-0.566**
				(-5.786)		(-3.345)
Rel-Size				-0.056		0.039
				(-1.395)		(0.873)
Coverage					-0.006	
					(-1.758)	
Q						-0.604**
						(-3.362)
AGE					-0.024**	-0.024**
					(-3.064)	(-2.966)

Table III
Marginal Effects

In this table we investigate the economic significance of each variables. Marginal effects quantify how much the probability of default changes, everything else held constant, when one of the variables changes by some discrete amount. The marginal effect is given by the partial derivative of the probability of default with respect to a variable multiplied by the variable change. The logit and duration marginals are given by

$$\Delta\hat{P}(y=1|x) = \hat{\beta}\frac{\exp(\bar{x}\hat{\beta})}{[1+\exp(\bar{x}\hat{\beta})]^2}\Delta x \text{ and } \Delta\hat{P}(y=1|x) = \frac{\partial h}{\partial x}\exp[-h(\bar{x},\hat{\beta})]\Delta x$$

respectively, where $h(x,\beta)$ is the hazard rate. For both logit and duration models the partial derivatives of the default probability depend on the value of the variables. One natural choice is to evaluate it at the mean value, and to consider a change in the level equal to one standard deviation.

	Δx	Δx %	Ohl	Zmi	Shu	Pvt	Dur
WC/TA	0.19	0.88	0.00			0.00	
RE/TA	0.35	2.64				0.04	0.02
OIA/TA	0.09	0.79				-0.05	-0.04
S/TA	0.70	0.59				-0.00	
log(TA)	1.99	0.35	-0.04			-0.02	
TL/TA	0.17	0.29	0.23	0.35	0.32	0.23	0.21
CA/CL	1.24	0.60	0.02	0.03		0.02	
NI/TA	0.09	4.16	0.01	-0.03	-0.01	0.01	
fund/TA	0.24	0.99	-0.01			0.00	
INTWO	0.34	2.51	0.06			0.04	0.03
CHIN	0.57	3.43	-0.19			-0.19	-0.14
Sigma	0.06	0.55			0.05		0.04
Diff-Ret	0.47	25.32			-0.16		-0.07
Rel-Size	2.00	0.20			-0.04		0.02
Coverage	0.88	1.89				-0.00	
Q	0.83	0.55					-0.13
AGE	11.87	0.76				-0.08	-0.07

Table IV
Out-of-Sample Forecasts

This tables show several measures of the model's ability to forecast default. Panel A of Table IV reports the *Total Performance*measures: in the four columns on the left side of the table we present the percentages of out-of-sample correct and incorrect classifications for *Default* and *Non-Default* cases. In the rightmost column we report the ECM, that represents the cost of incorrect classification, measured as the number of basis points per dollar invested.

$$ECM = p_I P(II|I)c_I + p_{II} P(I|II)c_{II},$$

where c_I is the cost of classifying a non-default firm as a default firm (type I error), and c_{II} represents the cost of classifying a default firm as a non-default firm (type II error). We choose $c_I = 1\%$ and fix $c_{II} = 40\%$. p_I and p_{II} are the unconditional probabilities of the two groups (group I is the healthy firm group; group II the default group). For estimates, we use the empirical frequencies observed year by year. $P(II|I)$ is the probability that the model classifies a firm which is really group I as belonging to group II, and $P(I|II)$ is the probability that the model classifies a firm which is really group II as belonging to group I. Panel B reports the decile percentage of correct classification and the AR. To construct the AR, we count how many firms actually default within the $\alpha\%$ of firms with the highest probability of default, $0 \leq \alpha \leq 1$. We then plot this cumulative percentage of correct classifications on the unit interval, obtaining a curve that takes the value 0 at $\alpha = 0$ and the value 1 at $\alpha = 1$. This curve is called the Cumulative Accuracy Profile (CAP). A theoretically *perfect model* would predict all defaults in the first $\hat{\alpha}\%$ of firms, where $\hat{\alpha}$ is equal to the frequency of defaults in the sample. The plot of the perfect model would be a straight line for all $\alpha < \hat{\alpha}$ and would be equal to 1 for $\alpha \geq \hat{\alpha}$. AR is defined as the ratio of the integral above the 45 degree line of the model's CAP to the same integral for the perfect model.

Panel A: Total Performance

	Non-Default		Default		ECM
	correct	incorrect	correct	incorrect	
Alt	66.4	33.6	59.7	40.3	45.8
Ohl	85.4	14.6	56.5	43.5	27.9
Zmi	81.9	18.1	50.0	50.0	33.3
Shu	85.9	14.1	63.8	36.2	25.3
Pvt	86.8	13.2	51.6	48.4	28.0
Dur	89.3	10.7	67.2	32.8	20.8

Panel B: Default Classification Accuracy

Decile	Alt	Ohl	Zmi	Shu	Pvt	Dur
1	19.4	46.8	35.5	50.0	50.0	62.1
2	16.1	17.7	16.1	20.7	16.1	13.8
3	21.0	6.5	17.7	17.2	9.7	3.4
4	8.1	12.9	11.3	0.0	6.5	8.6
5	12.9	3.2	4.8	5.2	6.5	6.9
6-10	22.6	12.9	14.5	6.9	11.3	5.2
AR	0.359	0.597	0.518	0.686	0.605	0.698

Table V
Out-of-Sample Forecasts

This table reports robustness check results. We repeat our estimation procedure many times, each time moving the validation sample ahead one year. We start by estimating the models using the observations belonging to the years before 1982 as estimation sample, and the observations after 1982 (included) as the validation group. Next, we use observations belonging to the years before 1983 to estimate the models, and those after 1983 to form out-of-sample predictions, and so on. We report out-of-sample ECMs and ARs (left and right side respectively).

	ECM						AR					
Year	Alt	Ohl	Zmi	Shu	Pvt	Dur	Alt	Ohl	Zmi	Shu	Pvt	Dur
1982	55.2	32.3	33.8	34.2	32.3	31.9	0.535	0.591	0.546	0.557	0.519	0.589
1983	51.6	31.4	33.6	33.4	32.9	32.5	0.557	0.593	0.553	0.596	0.502	0.570
1984	50.4	31.5	34.0	33.8	32.0	32.2	0.532	0.588	0.554	0.587	0.558	0.590
1985	47.4	30.8	33.7	32.3	30.0	29.0	0.559	0.597	0.544	0.611	0.599	0.647
1986	47.7	30.5	33.8	31.0	30.4	27.0	0.536	0.591	0.526	0.607	0.593	0.650
1987	45.8	30.1	33.9	30.7	30.3	27.8	0.527	0.565	0.504	0.588	0.565	0.624
1988	45.4	29.5	33.9	29.7	30.4	26.9	0.540	0.560	0.482	0.591	0.559	0.632
1989	43.3	28.1	33.0	27.7	28.2	25.0	0.532	0.556	0.484	0.615	0.563	0.653
1990	42.8	26.8	32.1	26.6	26.5	23.8	0.546	0.550	0.485	0.618	0.569	0.658
1991	42.7	26.6	32.1	25.6	25.9	23.6	0.551	0.550	0.484	0.626	0.574	0.653
1992	40.8	26.9	31.8	26.0	25.6	24.3	0.541	0.552	0.495	0.638	0.573	0.654
1993	42.0	26.8	33.0	27.2	26.2	23.7	0.540	0.558	0.489	0.638	0.580	0.667
1994	43.7	26.6	33.3	27.4	26.6	23.6	0.525	0.578	0.499	0.655	0.600	0.687
1995	44.9	27.6	34.1	28.1	27.2	23.3	0.513	0.612	0.517	0.674	0.634	0.716
1996	46.8	27.9	33.9	27.2	27.6	22.0	0.468	0.640	0.549	0.697	0.655	0.736
1997	45.8	27.9	33.3	25.3	28.0	20.8	0.359	0.597	0.518	0.686	0.605	0.698
1998	46.5	24.5	29.4	21.6	22.8	18.5	0.401	0.659	0.584	0.731	0.690	0.763
1999	43.9	19.5	23.9	15.8	17.5	14.1	0.369	0.638	0.607	0.838	0.644	0.800

Table VI
Default Decile Portfolio Returns

This table reports the returns on the 10 portfolios for each model, before and after risk adjusting by a 4-factor model: the Fama and French (1993) three factor model plus a momentum factor. ** and * indicate statistically significance at the 1% and 5% statistical level, respectively.

Value-Weighted Returns

	Low	2	3	4	5	6	7	8	9	High
Alt	1.50	1.20	1.25	1.17	1.23	1.17	0.91	1.17	1.03	1.26
Ohl	1.34	0.91	1.03	1.18	1.40	1.10	1.44	1.34	1.30	0.87
Zmi	1.56	1.16	1.22	1.23	1.31	1.13	1.15	1.26	1.03	1.25
Shu	1.44	1.09	1.14	1.22	1.33	1.17	1.19	1.20	1.21	1.11
Pvt	1.25	1.03	1.05	1.25	1.22	1.39	1.33	1.30	1.16	1.05
Dur	1.16	1.11	1.17	1.12	1.31	1.41	1.40	1.19	1.13	1.44

Risk-Adjusted Returns

	Low	2	3	4	5	6	7	8	9	High
Alt	0.59**	0.12	0.10	0.01	0.11	-0.15	-0.26*	-0.09	-0.23*	0.02
	(4.61)	(0.85)	(0.67)	(0.06)	(0.88)	(-1.26)	(-2.06)	(-0.82)	(-1.96)	(0.04)
Ohl	0.51*	-0.18	0.05	0.04	0.12	-0.10	0.25*	-0.00	-0.03	-0.30
	(2.59)	(-1.15)	(0.43)	(0.26)	(1.09)	(-1.03)	(2.08)	(-0.02)	(-0.21)	(-1.57)
Zmi	0.66**	0.24	0.18	0.09	0.14	-0.06	-0.23*	0.04	-0.32*	-0.13
	(3.81)	(1.74)	(1.50)	(0.80)	(1.46)	(-0.52)	(-1.98)	(0.31)	(-2.33)	(-0.60)
Shu	0.59**	0.13	0.06	0.04	0.03	0.04	-0.14	-0.21	-0.18	-0.15
	(3.10)	(1.08)	(0.53)	(0.32)	(0.18)	(0.31)	(-1.24)	(-1.49)	(-1.11)	(-0.81)
Pvt	0.33	-0.01	-0.08	0.15	0.06	0.19	0.10	-0.04	-0.16	-0.16
	(1.93)	(-0.08)	(-0.58)	(1.56)	(0.55)	(1.42)	(0.78)	(-0.30)	(-1.00)	(-0.81)
Dur	0.30	0.05	0.12	-0.05	0.03	0.15	0.04	-0.16	-0.17	-0.03
	(1.53)	(0.31)	(0.94)	(-0.36)	(0.20)	(1.24)	(0.43)	(-1.11)	(-1.11)	(-0.13)

Table VII
Characteristics of Default Decile Portfolios

The top part of the table reports the average characteristics of the decile portfolios when we use the probability of default obtained from the Dur model. For each decile portfolio we run a time-series regression of the value-weighted returns on a four-factor model. The bottom part of the table reports loading estimates along with robust Newey and West (1987) t-statistics. ** and * indicate statistically significance at the 1% and 5% statistical level, respectively. *a) in millions of dollars. b)in millions of shares.*

	Low	2	3	4	5	6	7	8	9	High
$\overline{ret}(VW)$	1.16	1.11	1.17	1.12	1.31	1.41	1.40	1.19	1.13	1.44
$\sigma(VW)$	6.15	4.90	4.47	4.42	4.32	3.87	3.98	4.28	4.70	5.56
$\overline{size^a}$	3098.7	2973.8	2482.2	1868.6	1934.6	1831.9	1504.5	1102.0	793.2	357.9
$\overline{BM}$	0.578	0.497	0.598	0.627	0.668	0.712	0.756	0.782	0.848	0.849
$\overline{volume^b}$	464.8	279.8	186.7	142.1	145.7	121.7	97.5	79.0	58.3	33.0
α	0.003	0.001	0.001	-0.001	0.001	0.001	0.001	-0.002	-0.001	-0.001
	(1.529)	(0.309)	(0.941)	(-0.358)	(0.199)	(1.243)	(0.431)	(-1.112)	(-1.112)	(-0.129)
β_{mkt}	0.972**	0.953**	0.947**	0.994**	0.949**	0.899**	0.947**	1.011**	1.081**	1.257**
	(12.4852)	(19.409)	(24.751)	(24.190)	(30.019)	(29.213)	(27.352)	(27.536)	(34.452)	(26.629)
β_{SMB}	-0.156*	-0.188**	-0.047	-0.078	-0.024	-0.106	-0.089	0.135*	0.164**	0.411**
	(-2.1013)	(-3.274)	(-0.971)	(-1.405)	(-0.525)	(-1.867)	(-1.471)	(2.358)	(3.821)	(5.308)
β_{HML}	-0.623**	-0.176	0.021	0.182*	0.184*	0.313**	0.403**	0.479**	0.427**	0.737**
	(-6.7341)	(-1.927)	(0.246)	(2.532)	(2.444)	(4.401)	(5.315)	(5.582)	(4.679)	(5.406)
β_{mom}	-0.086	-0.031	-0.102*	-0.088	0.090*	0.033	0.073	0.010	-0.076	-0.121
	(-1.782)	(-0.534)	(-2.098)	(-1.734)	(1.973)	(0.832)	(1.878)	(0.202)	(-1.412)	(-1.274)
R^2	0.816	0.828	0.847	0.848	0.819	0.829	0.843	0.812	0.806	0.751
$\overline{\beta_{SMB}}$	0.66	0.75	0.67	0.65	0.59	0.50	0.53	0.67	0.82	1.03
$\overline{\beta_{HML}}$	-0.36	-0.15	0.00	0.05	0.09	0.19	0.25	0.25	0.24	0.33

Table VIII
Fama-MacBeth Procedure

This table reports result of a two step Fama and MacBeth (1973) regression. In the first step, for each stock, at June of each year, we run a time-series regression using a 5-year window of monthly data to obtain estimates of the stock loadings on the factors. In the second step, at June of each year, we run a cross-sectional regression of the probability of default, P_{it}, on the individual stock factor loadings, $\hat{\beta}_{SMB,it}$ and $\hat{\beta}_{HML,it}$, and on some control variables (size and book-to-market):

$$P_{it} = \lambda_{0,t} + \lambda_{SMB,t}\ \hat{\beta}_{SMB,it} + \lambda_{HML,t}\ \hat{\beta}_{HML,it} + \lambda_{x,t}\ x_{it} + \epsilon_{it}$$

Then we compute time-series average and robust standard errors of the cross-sectional estimates

$$\hat{\lambda} = \frac{1}{T}\sum_{t=1}^{T}\hat{\lambda}_t, \qquad SE(\hat{\lambda}) = \frac{1}{T}\sum_{j=-\infty}^{\infty} cov_T\left(\hat{\lambda}_t, \hat{\lambda}_{t-j}\right)$$

** and * indicate one-side-test statistically significance at the 1% and 5% statistical level, respectively.

	(1)	(2)	(3)	(4)
λ_0	-6.031	-4.469	-6.001	-4.211
	(-71.622)	(-17.351)	(-79.956)	(-16.081)
λ_{MKT}			0.049	
			(1.247)	
λ_{SMB}	0.067**			-0.102
	(2.394)			(-4.968)
λ_{HML}	0.253**			0.219**
	(7.763)			(7.288)
λ_{Size}		-0.152		-0.165
		(-7.938)		(-8.062)
λ_{BM}		0.609		0.538
		(6.381)		(5.614)

Table IX
Factors Performance in Bad Financial Distres Years

In this table we report correlation coefficients between year-to-year changes of the aggregate default probability (ΔDEF), and the returns on SMB, and HML for the years 1961-2002, and the two subsamples 1961-1981 and 1982-2002. We compute the default probabilities for the years that are outside our original sample using the average of the 1983-2000 time-series of the hazard rate estimated paramters. The aggregate default probability is obtained by taking a simple value-weighted average of the firm probabilities. Moreover to account for the lag in availability of accounting information we compute returns on the factors from June to May of each year.

	1961-2002		*1961-1981*		*1982-2002*	
	SMB	HML	SMB	HML	SMB	HML
corr(ΔDEF,factor)	17.99%	-35.00%	26.91%	-26.55%	-0.28%	-43.14%
	(1.166)	(-2.268)	(1.233)	(-1.216)	(-0.013)	(-1.977)
mean	0.0248	0.0563	0.0481	0.0658	0.0014	0.0468
$\Delta DEF > 0$	0.0205	0.0385	0.0641	0.0348	0.0056	0.0221
$\Delta DEF > q(75)$	0.0551	0.0235	0.0071	0.0382	0.0123	-0.0698
$\Delta DEF > q(90)$	0.0273	-0.0450	0.1029	-0.0010	0.0251	-0.0768
$\Delta DEF < 0$	0.0279	0.0697	0.0305	0.0998	-0.0032	0.0740
$\Delta DEF < q(75)$	0.0153	0.0666	0.0609	0.0744	-0.0020	0.0833
$\Delta DEF < q(90)$	0.0245	0.0670	0.0424	0.0728	-0.0011	0.0598
$\Delta[0]$	-0.0074	-0.0312	0.0336	-0.0650	0.0089	-0.0519
$\Delta[q(75)]$	0.0398	-0.0430	-0.0538	-0.0362	0.0143	-0.1531
$\Delta[q(90)]$	0.0028	-0.1120	0.0605	-0.0738	0.0262	-0.1366
$\Delta DEF < q(25)$	0.0047	0.0998	0.0112	0.1268	0.0036	0.0850
$\Delta DEF < q(10)$	-0.0103	0.1243	-0.0115	0.0606	-0.0090	0.1880
$\Delta[q(75) - q(25)]$	0.0504	-0.0763	-0.0041	-0.0886	0.0087	-0.1548
$\Delta[q(90) - q(10)]$	0.0376	-0.1693	0.1143	-0.0616	0.0341	-0.2647

Table X
Aggregate Default Risk and HML

This table reports results about our investigation of the relation between default risk and HML using quarterly data for the period 1969-2002. In roder to account for all possible model misspecifications we run several versions of the basic model

$$HML_t = \beta_0 + \beta_1 \Delta DEF_t + \beta_2 x_t + \mu_t$$

x_t is a vector of control variable that includes: change in credit spreads, the marekt return, the return on SMB, a recession dummy, and an interaction variable between the recession dummy and ΔDEF_t. Model (2) is estimated using Hansen (1982) efficient GMM, instrumenting ΔDEF_t with the change in the aggregate Tobin's Q and with ΔDEF_{t-1}. We obtain the other models parameters using OLS and we compute Newey and West (1987) robust standard errors, using 2 leads and 2 lags. ** and * indicate statistically significance at the 1% and 5% statistical level, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	1969-2002						*69-85*	*86-02*
const	0.004*	0.004*	0.004*	0.008**	0.008**	0.007**	0.008**	0.007*
	(2.167)	(2.029)	(2.198)	(4.170)	(4.497)	(3.627)	(3.821)	(2.563)
ΔDEF_t	-6.436	-15.722**			-9.450**	-7.383*	-15.306**	-6.901*
	(-1.803)	(-2.914)			(-3.193)	(-2.276)	(-2.998)	(-2.013)
$\Delta Spread_t$			-0.191		-1.014**	-1.069**	-0.663	-1.793**
			(-0.492)		(-3.321)	(-3.615)	(-1.917)	(-2.999)
MKT_t				-0.004**	-0.004**	-0.004**	-0.003**	-0.005**
				(-6.285)	(-6.677)	(-6.396)	(-4.027)	(-5.142)
SMB_t				0.063	-0.003	-0.034	-0.085	0.003
				(0.668)	(-0.038)	(-0.405)	(-0.907)	(0.022)
Rec_t						0.004		
						(1.081)		
$Rec_t * \hat{\epsilon}_t^D$						-11.325		
						(-1.861)		
R^2	0.015		-0.006	0.249	0.317	0.322	0.254	0.390